Exam. Code : 103206
Subject Code : 1192

B.A./B.Sc. 6th Semester

MATHEMATICS (Numerical Analysis)

Paper—II

Time Allowed—Three Hours] [Maximum Marks—50

Note :—Do any **FIVE** questions, selecting at least **TWO** questions from each section. All questions carry equal marks. Non-programmable scientific calculator is allowed.

SECTION—A

1. (a) If $r = 3h(h^6 - 2)$, find the percentage error in r at $h = 2.1$ if the error in h is 6.5%.

   (b) Apply Bisection method in four stages to find the root of the equation $x^3 - 4x - 9 = 0$.

2. (a) Show that Newton's method is of quadratic convergence. Estimate the value of $\sqrt{\frac{1}{41}}$ up to four decimal places by applying Newton's iterative method.

   (b) Apply Gauss elimination method to solve the system of equations :

   $X + Y + Z = 9;$
   $2X - 3Y + 4Z = 13;$
   $3X + 4Y + 5Z = 40$

3. (a) Apply triangular method to solve the system of equations :

$$3X + 2Y + 7Z = 4;$$
$$2X + 3Y + Z = 5;$$
$$3X + 4Y + Z = 7.$$

(b) Solve by Jacobi's iteration method, the equations :

$$20X + 4Y - 2Z = 17;$$
$$3X + 20Y - Z = -18;$$
$$2X - 3Y + 20Z = 25.$$

4. (a) Assuming that the following values of y belong to the polynomial of degree 4, compute the next three values.

x	0	1	2	3	4	5	6	7
y	1	–1	1	–1	1	–	–	–

(b) Prove the following identities :

(i) $\nabla = 1 - E^{-1}$

(ii) $\delta = E^{\frac{1}{2}} - E^{\frac{-1}{2}}$.

5. (a) Prove with the usual notations that :

(i) $(E^{\frac{1}{2}} + E^{\frac{1}{2}})(1+\Delta)^{\frac{1}{2}} = 2 + \Delta$

(ii) $\Delta^3 y_2 = \nabla^3 y_5$.

(b) Evaluate :

(i) $\Delta^2\left(\frac{5x+12}{x^2+5x+16}\right)$ and

(ii) $\Delta^n(e^x)$.

SECTION—B

6. (a) Using Newton's interpolation formula find the cubic polynomial which takes the following values. Hence evaluate f(4).

x	0	1	2	3
f(x)	1	2	1	10

(b) Apply Bessel's formula to obtain y_{25}, given $y_{20} = 2854$, $y_{24} = 3162$, $y_{28} = 3544$, $y_{32} = 3992$.

7. (a) Using Lagrange's interpolation formula find the value of y, when x = 10, if the following values of x and y are given :

x	5	6	9	11
y	12	13	14	16

(b) Determine f(x) as a polynomial in x for the following data, using Newton's divided difference formula.

x	−4	−1	0	2	5
f(x)	1245	33	5	9	1335

8. (a) A function is given according to the table below. Find the derivation for x = 0.5.

x	0.35	0.40	0.45	0.50	0.55	0.60	0.65
y	1.521	1.506	1.488	1.467	1.444	1.418	1.389

(b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using (i) Simpson's 1/3rd rule taking h = 1/4 and (ii) Simpson's 3/8th rule taking h = 1/6.

9. (a) Find by Taylor's series method the value of y at x = 0.1 and x = 0.2 to five decimal places from $\frac{dy}{dx} = x^2y - 1$, y(0) = 1.

(b) Using Runge-Kutta method of fourth order solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with y(0) = 1 at x = 0.2, 0.4.

10. (a) Find the value of y for x = 0.1, by Picard's method given that $\frac{dy}{dx} = \frac{y - x}{y + x}$ with y(0) = 1.

(b) Using Milne's Predictor-Corrector method to obtain the solution of the equation $\frac{dy}{dx} = x - y^2$ at x = 0.8, given that y(0) = 0.0000, y(0.2) = 0.0200, y(0.4) = 0.0795, y (0.6) = 0.1762.